Math for Finance Professionals

Course Introduction

Learning Objectives

- Explain the difference between simple and compound interest
- Calculate and compare effective and nominal interest rates
- Build a range of discounted cash flow models that represent real-world applications
- Calculate the price and yield of annual bonds
- Analyze a range of financial market scenarios using key financial statistics

Simple and Compound Interest

CFI

Simple Interest

Interest is the fee that a borrower pays to borrow an amount of money called the principal.

Interest Payment = **Principal** × **Interest Rate**

With simple interest, the principal does not change, so the interest payments do not change.

Simple Interest

Interest is the fee that a borrower pays to borrow an amount of money called the principal.

Interest Payment = **Principal** × **Interest Rate**

+ $5   + $5   + $5

$100	$105	$110	$115
Year 0	Year 1	Year 2	Year 3

Example

Principal = **$100**

Interest Rate = **5%** per annum

Total Periods = **3** years

Interest Payment = **$100** × **5%**

= **$5**

Time Value of Money

The value of a dollar **today** is not the same as the value of a dollar **in the future**.

What would you rather have?

$10,000
today

$10,000
a year from now

Time Value of Money

Opportunity Cost
Defer from consuming or investing that money.

Inflation
You can purchase less in the future with that money.

Default Risk
Someone may default on money they owe you.

What would you rather have?

$10,000
today

$10,000
a year from now

Calculating FV Using Simple Interest

$$FV = 100 + (100 \times 5\% \times 3)$$

$$FV = PV + (PV \times i \times n)$$

$$FV = PV \times (1 + i \times n)$$

FV: Future Value

PV: Present Value

i: Interest Per Period

n: Number of Periods

Calculating PV Using Simple Interest

$$FV = PV \times (1 + i \times n)$$

$$115 = PV \times (1 + 5\% \times 3)$$

FV: Future Value

PV: Present Value

i: Interest Per Period

n: Number of Periods

Calculating PV Using Simple Interest

$$PV = FV \times \frac{1}{(1 + i \times n)} \quad \text{OR} \quad PV = \frac{FV}{(1 + i \times n)}$$

FV: Future Value

PV: Present Value

i: Interest Per Period

n: Number of Periods

Calculating PV Using Simple Interest

$$PV = FV \times \frac{1}{(1 + i \times n)} \quad \text{OR} \quad PV = \frac{FV}{(1 + i \times n)}$$

FV: Future Value

PV: Present Value

i: Interest Per Period

n: Number of Periods

Example

If you are going to receive $100 five years from now, how much is that worth today, assuming 4% annual simple interest?

FV = **$100**

i = **4%**

n = **5 years**

$$PV = \$100 \times \frac{1}{(1 + 4\% \times 5)}$$

$$= \$83.33$$

Compound Interest

Simple Interest

Interest is calculated only on the principal amount.

Compound Interest

Interest is calculated on the principal plus accumulated interest.

Compound Interest

$$\text{Compound Interest Earned Each Period} = (\text{Principal} + \text{Previously earned interest}) \times \text{Rate of Interest}$$

Example

Principal = $100

Interest Rate = 5% per annum

Total Periods = 3 years

Year 0	Year 1	Year 2	Year 3
$100	$105	$110.25	$115.76
	+ ($100.00 × 5%) = $5.00	+ ($100.00 × 5%) = $5.25	+ ($100.00 × 5%) = $5.51

Calculating FV Using Compound Interest

	× (1 + 5%)	× (1 + 5%)	× (1 + 5%)
$100	$105	$110.25	$115.76
Year 0	Year 1	Year 2	Year 3

Present Value (PV) → **Future Value (FV)**

Future Value (Compounding)

Example

PV = $100

i = 5%

n = 3 years

$$FV = \$100 \times (1 + 5\%)^3$$

$$FV = PV \times (1 + i)^n$$

Calculating PV Using Compound Interest

÷ (1 + 5%) ÷ (1 + 5%) ÷ (1 + 5%)

$100	$105	$110.25	$115.76
Year 0	Year 1	Year 2	Year 3

Present Value (PV) ← **Future Value (FV)**

Present Value (Discounting)

Example

FV = $115.76

i = 5%

n = 3 years

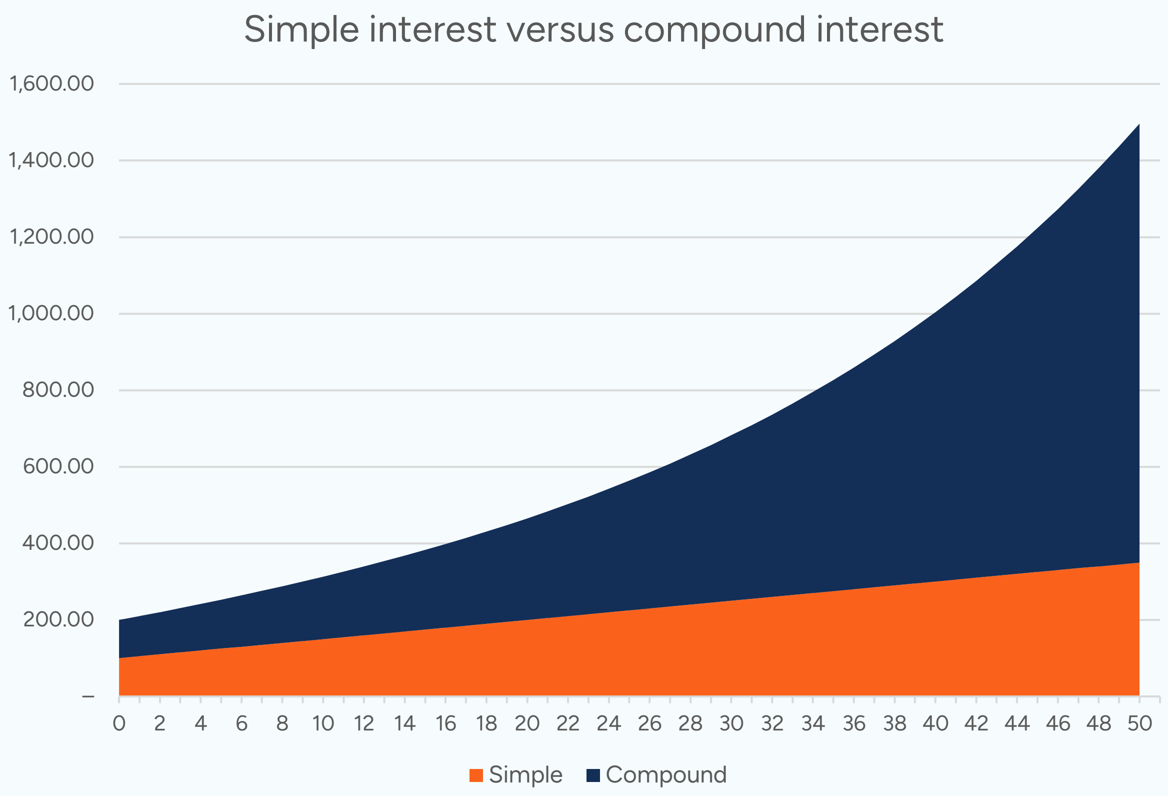
$$PV = \frac{115.76}{(1 + 5\%)^3} = \$100$$

$$PV = \frac{FV}{(1 + i)^n} \quad \text{OR} \quad PV = FV \times \frac{1}{(1 + i)^n}$$

Simple Interest vs. Compound Interest

Compound interest is the eighth wonder of the world.

He who understands it, earns it.
He who doesn't, pays it.



Nominal and Effective Interest Rates

Changing Compounding Periods – Part 1

The **nominal interest rate** is **10%**.

Today								12 months
$1,000.00	× (1 + **10%**) →							**$1,100.00**
$1,000.00	× (1 + **5%**) →			**$1,050.00**	× (1 +**5%**) →			**$1,102.50**
$1,000.00	× (1 + **2.5%**) →	**$1,025.00**	× (1 + **2.5%**) →	**$1,050.63**	× (1 + **2.5%**) →	**$1,076.89**	× (1 + **2.5%**) →	**$1,103.81**

Changing Compounding Periods – Part 2

Imagine you have **$1,000** to invest today.

Which option would you choose?

Bank A	**Bank B**
Annual interest rate of 10% compounded **quarterly**.	Annual interest rate of 10% compounded **monthly**.

Changing Compounding Periods – Part 2

The **nominal** interest rate is **10%.**

Compounding Frequency	Present Value	Frequency	Future Value
Annual	$1,000.00	1	1,100.00
Semi Annual	$1,000.00	2	1,102.50
Quarterly	$1,000.00	4	1,103.81
Monthly	$1,000.00	12	1,104.71
Weekly	$1,000.00	52	1,105.06
Daily	$1,000.00	365	1,105.16

! **Increasing** the compounding frequency **increases** the future value.

Calculating FV When n = 1

Calculation

$$FV = PV \times \left(1 + \frac{i}{f}\right)^{n \times f}$$

$$= \$1{,}000 \times \left(1 + \frac{10\%}{4}\right)^{1 \times 4}$$

$$= \$1{,}103.81$$

Example

Present Value (PV) = $1,000

Interest Rate (i) = 10%

Frequency (f) = 4

Number of Years (n) = 1 year

Calculating FV When n > 1

Future Value Example

Present Value (PV) = $1,000

Interest Rate (i) = 10%

Frequency (f) = 4

Number of Years (n) = 3 years

Calculation

$$FV = PV \times \left(1 + \frac{i}{f}\right)^{n \times f}$$

$$= \$1{,}000 \times \left(1 + \frac{10\%}{4}\right)^{3 \times 4}$$

$$= \$1{,}344.89$$

Calculating PV When f > 1 and n > 1

Present Value (Discounting) Example

Future Value (FV) = $1,500

Interest Rate (i) = 6%

Frequency (f) = 2

Number of Years (n) = 4 years

Calculation

$$PV = FV \times \frac{1}{\left(1 + \frac{i}{f}\right)^{n \times f}}$$

$$PV = \$1500 \times \frac{1}{\left(1 + \frac{6\%}{2}\right)^{4 \times 2}}$$

$$\boxed{PV = \$927.17}$$

Calculating PV When f > 1 and n > 1

Present Value Example

Future Value (FV) = $1,250

Interest Rate (i) = 10%

Frequency (f) = 12

Number of Years (n) = 3 years

Calculation

$$PV = FV \times \frac{1}{\left(1 + \frac{i}{f}\right)^{n \times f}}$$

$$PV = \$1250 \times \frac{1}{\left(1 + \frac{10\%}{12}\right)^{3 \times 12}}$$

$$\boxed{PV = \$1{,}184.11}$$

Effective Rate vs. Nominal Rate

Imagine you have **$1,000** to invest today.

Which option would you choose?

Bank A

Annual interest rate of **10%** compounded **quarterly**.

Bank B

Annual interest rate of **10%** compounded **monthly**.

Effective Rate vs. Nominal Rate

Simple

Nominal Rate

Annual Percentage Rate (APR)

- The interest rate has not taken into account any compounding.
- When **borrowing funds**, a bank advertises a **nominal rate, as this will typically be lower than the effective rate** (i.e., the actual cost of borrowing).

Compound

Effective Rate

Effective Annual Rate (EAR)

Annual Percentage Yield (APY)

- The interest rate has taken compounding into consideration.
- When **depositing** your funds, a bank advertises an **effective rate, as this will typically be higher than the nominal rate.**

Calculating The Effective Interest Rate

The **nominal interest rate** is **10%** and the investment period is **1 year.**

Compounding Frequency	Present Value	Frequency	Future Value
Annual	$1,000.00	1	1,100.00
Semi Annual	$1,000.00	2	1,102.50
Quarterly	$1,000.00	4	1,103.81
Monthly	$1,000.00	12	1,104.71
Weekly	$1,000.00	52	1,105.06
Daily	$1,000.00	365	1,105.16

! **Increasing** the compounding frequency **increases** the future value.

Converting Effective and Nominal Interest Rates

Convert Nominal Rate to Effective Rate

Nominal Interest Rate (r_{nom}) = 10.00%

# of Compounding Periods (f) = 4

$$r_{eff} = \left(1 + \frac{r_{nom}}{f}\right)^{f} - 1$$

$$= 10.38\%$$

Convert Effective Rate to Nominal Rate

Effective Annual Rate (r_{eff}) = 10.38%

# of Compounding Periods (f) = 4

$$r_{nom} = \left((1 + r_{eff})^{\frac{1}{f}} - 1\right) \times f$$

$$= 10.00\%$$

The Language of Compounding

01	Compounding period not given	Effective Rate	**5%** per year **1%** per month
02	Compounding period given, nominal vs. effective not stated	Nominal Rate	**8%** per year, compounded semi-annually
03	Interest rate stated as effective rate	Effective Rate	Effective **5%** per year compounded monthly

Power of Compounding

Example: **5% Nominal Rate** would equal: $r_{eff} = (1 + \frac{r_{nom}}{f})^f - 1$

Compounding	Effective Rate Formula	Effective Rate
Compounded annually	$(1+\frac{0.05}{1})^1 -1$	5%
Compounded semi-annually	$(1+\frac{0.05}{2})^2 -1$	5.06%
Compounded quarterly	$(1+\frac{0.05}{4})^4 -1$	5.09%
Compounded monthly	?	?
Compounded daily	$(1+\frac{0.05}{365})^{365} -1$	5.13%

Discounted Cash Flow Applications

Discounted Cash Flow Overview

Investors often need to **value an asset** that will generate **a single cash flow** or **a series of cash flows** in the future.

Stocks

Bonds

Derivatives

Company

Discounted Cash Flow Overview

Discounted Cash Flows (DCF)

DCF can help investors decide what should they pay **today** in order to generate an **adequate return**.

Equipment

Retirement Income

Loan

Discounting a Single Cash Flow

Example: If you are going to receive **$100** five years from now, how much is that worth today, assuming a **4%** annual compounding interest?

Discounting: Reducing the future cash flows to find their present values.

	÷ (1 + 4%)	÷ (1 + 4%)	÷ (1 + 4%)	÷ (1 + 4%)	÷ (1 + 4%)
$82.19	$85.481	$88.90	$92.46	$96.15	$100
Year 0	Year 1	Year 2	Year 3	Year 4	Year 5

Present Value (PV)

Future Value (FV)

$$\div (1 + 4\%)^5 \quad = \quad \times \frac{1}{\div (1 + 4\%)^5} = 0.8219$$

$$\$82.19 = \$100 \times 0.8219$$

$$PV = FV \times DF$$

Annuities Overview

Annuity

A **fixed amount of money** paid or received at **equal time periods** for a **fixed number of years**.

Home Mortgage

Retirement Income

Auto Loan

The Annuity Factor

Example

What is the present value of a $1,000 annuity for 3 years at 8% annual compounding?

Discount Factor = $\frac{1}{(1 + \mathbf{r})^{\mathbf{n}}}$

- **r**: rate per period
- **n**: number of periods

Annuity Factor: 2.577

Present Value: $1,000 x 2.577 = $2,577.10

Annuity Factor

The total of the discount factors in each period

- Interest rate (**r**)
- Number of cash flows (**n**)

Year	Discount Factor
1	$\frac{1}{(1+8\%)^1}$ = **0.9259**
2	$\frac{1}{(1+8\%)^2}$ = **0.8573**
3	$\frac{1}{(1+8\%)^3}$ = **0.7938**
Sum	**2.5771**

Auto Loan Example

Auto Loan

By Bus = 180 minutes

By Car = 40 minutes

Car price: $12,000

Down payment: $2,400 (20%)

Loan: $9,600

Rate: 8% APR compounded monthly

Term: 3 years (36 months)

What will your **monthly payments** be?

→ **Annuity**

Each of the 36 monthly payments will be exactly the same.

Using the PMT function

Auto Loan

Monthly auto loan payment, also known as the **annuity payment**.

→ Constant payment, which pays off the loan in full and pays the interest.

PMT Function

The Payment, or **PMT function**, will calculate the payment of an annuity.

Five Inputs

- ✓ Rate
- ✓ Number of Periods (NPER)
- ✓ Present Value
- ✓ Future Value
- ✓ Type

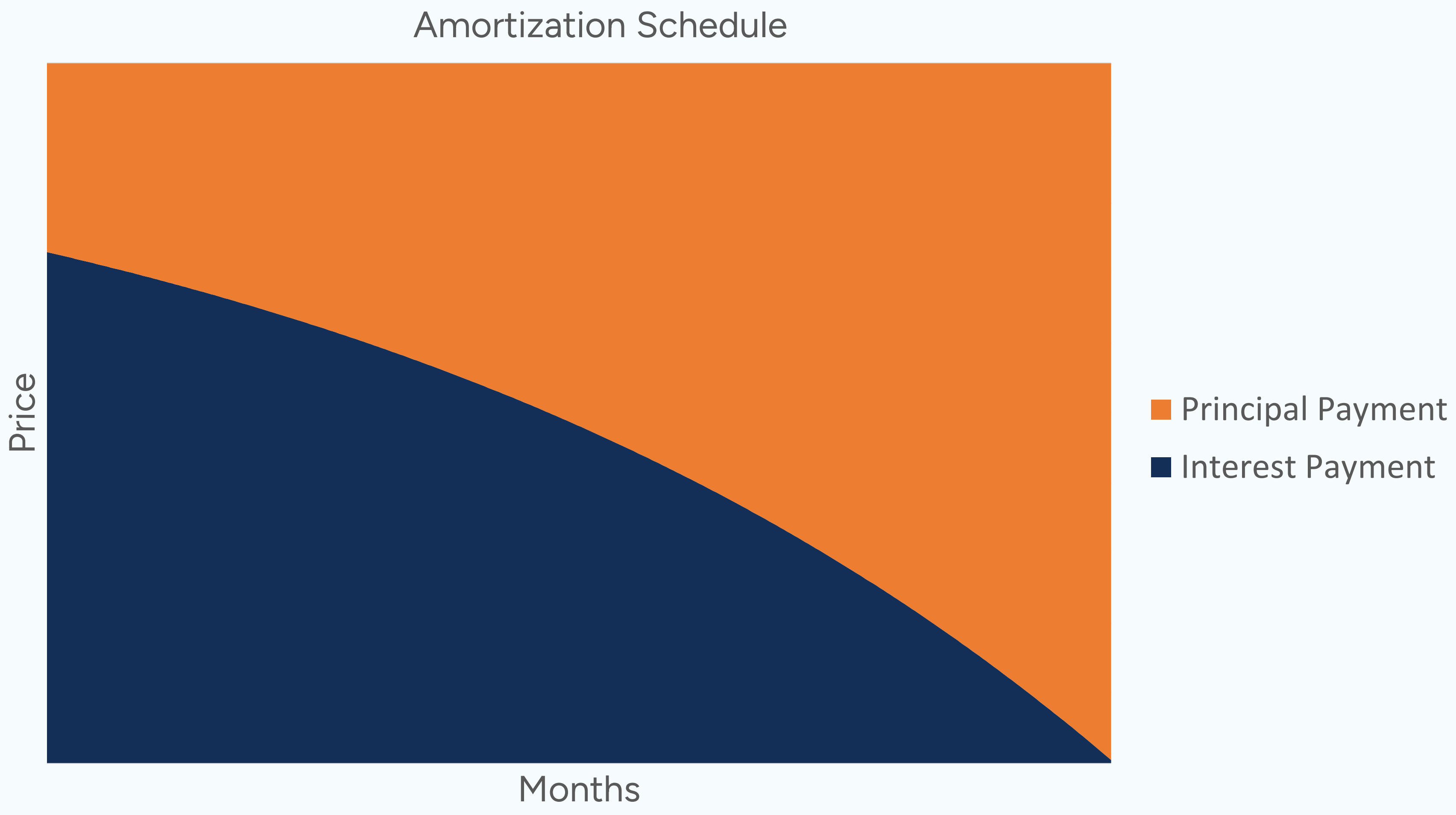
Mortgage Example

Key Inputs	
Purchase Price	$2,390,000.00
Down Payment	$717,000.00 (30%)
Mortgage Amount	$1,673,000.00
Interest Rate	5-year fixed at 5.25%
Mortgage Term	25 Years
Payment Frequency	Monthly
Compounding Period	Monthly

What will your **monthly payments** be?

Amortizing a Loan

A mortgage amortization schedule is a table that lists **each regular payment** on a mortgage **over time**.



Net Present Value (NPV)

Net Present Value (NPV) is the value of **all future cash flows** over the entire life of an investment discounted to the present **minus the initial investment**.

Example

- **Initial investment**: $1,000
- **Total period**: 3 Years
- **Annual cash flow**: $400
- **Discount rate**: 5%

Discount Factor $= \dfrac{1}{(1 + 5\%)^n}$

Year	0	1	2	3
Cash Flow (FV)	-1,000	400	400	400
Discount Factor		0.9524	0.9070	0.8638
PV Year 1	380.95			
PV Year 2	362.81			
PV Year 3	345.54			
NPV	**89.30**	**Positive NPV**		

NPV Decision Rule

Companies may look at the cost-benefit of a project by using the **NPV decision rule**.

Projects with a **positive NPV** return enough cash to more than cover the cost of the project.

PROCEED

Projects with a **negative NPV** fail to return enough cash to cover the cost of the project.

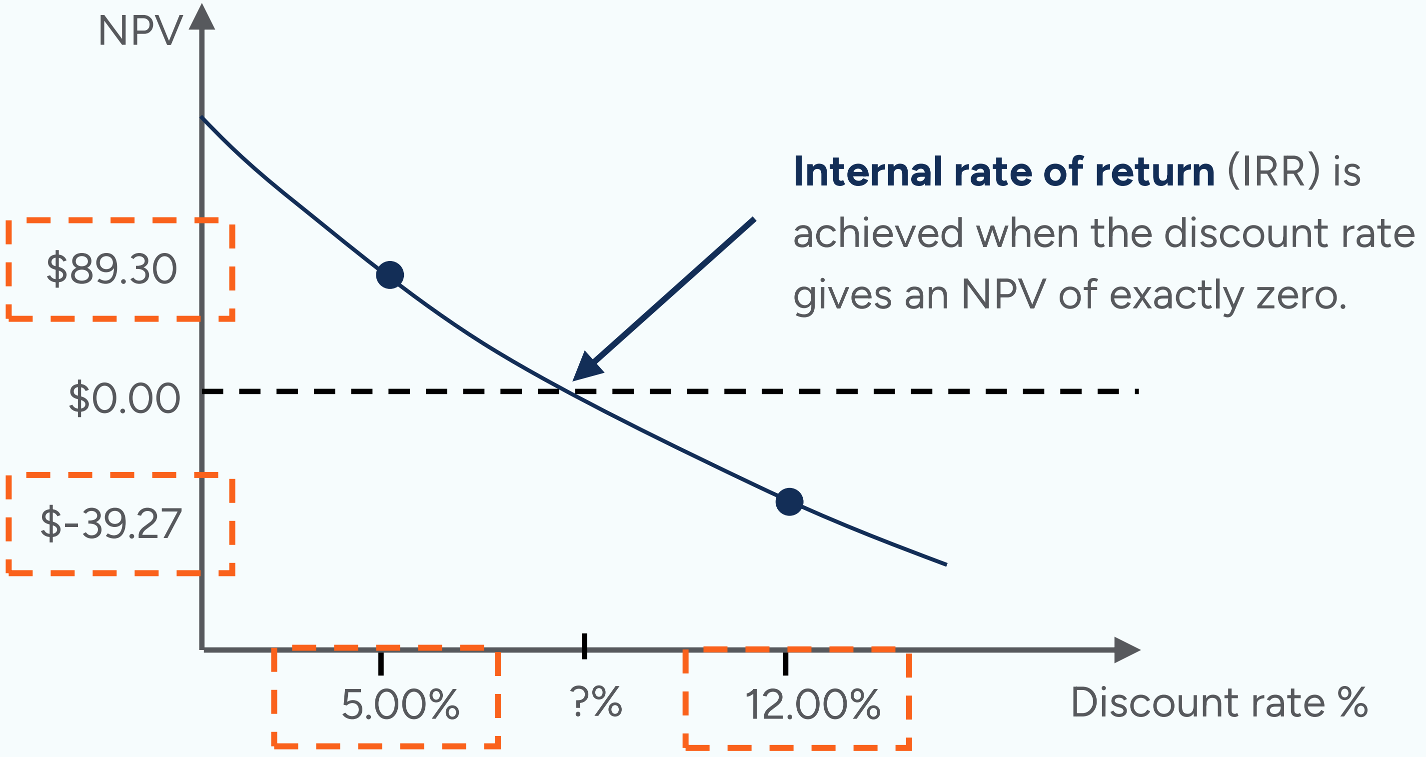
REJECT

Internal Rate of Return

As the **required return** (discount rate) increases, the **NPV falls**. To get a higher return, pay less for the investment.

Example

- **Initial investment**: $1,000
- **Total period**: 3 Years
- **Annual cash flow**: $400
- **Discount rate**: 5%
- **NPV**: $89.30



Bond Pricing

What Is a Bond

A **bond** is a **debt** security that allows an issuer, such as a company or government, to raise money from investors.

Issue Date: Jan 1, 2024

Maturity Date: Dec 31, 2028

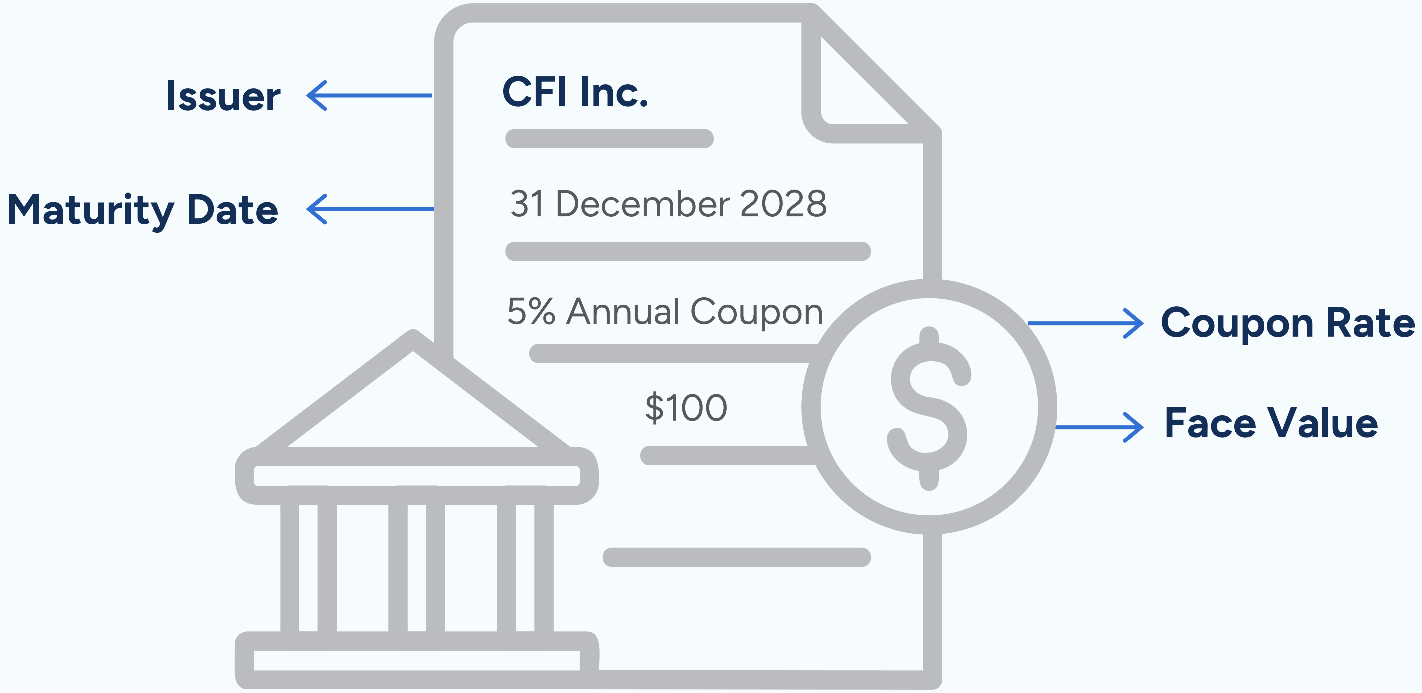
5-Year Bond

Face/Nominal/Par Value:

The amount repaid by the issuer on the maturity date.

Coupon = Par x Coupon Rate

$100 x 5% = $5



Pricing a Bond

If a bond investor was to sell the bond before the maturity date, what would the price be?

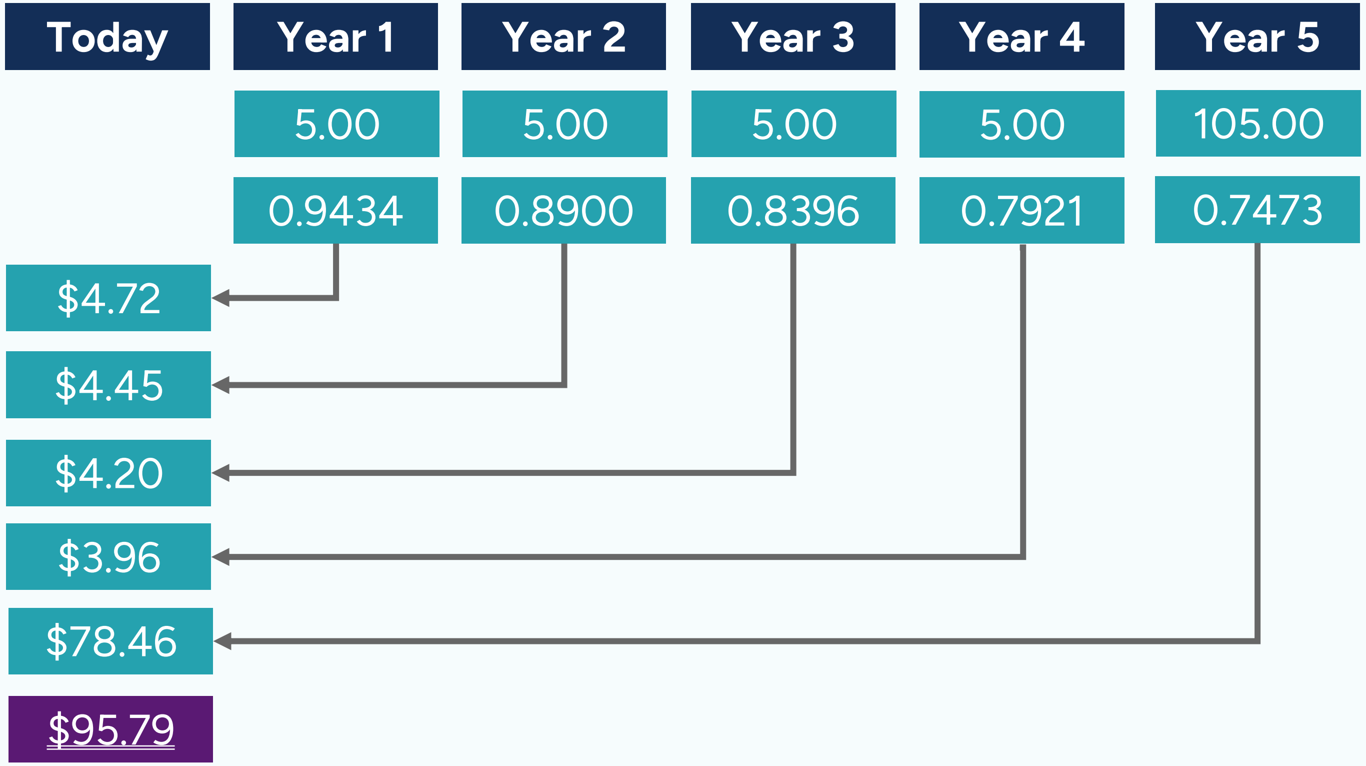
The **price** of the bond is the sum of the **present values** of the **future cash flows** of the bond.

Discount Factor

i: Yield to Maturity = 6%

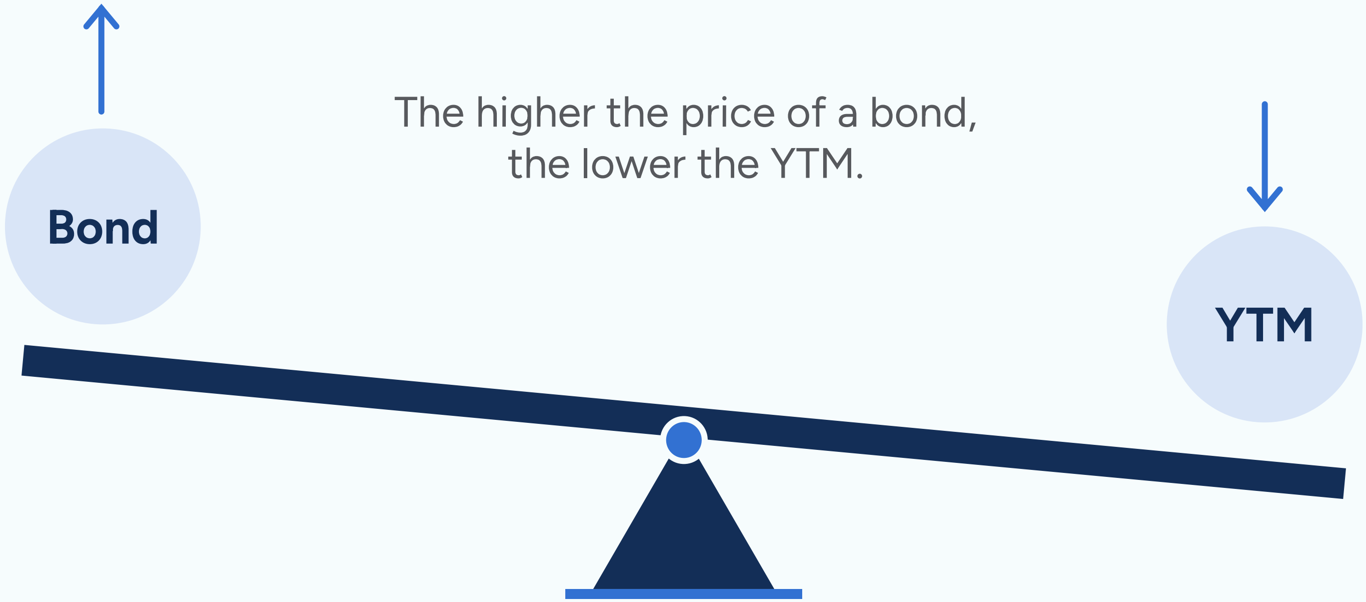
n: Year

$$= \frac{1}{(1 + 6\%)^{n}}$$



The Relationship Between Price and YTM

It is critical to understand the **relationship** between the price of a **bond** and the **yield to maturity (YTM)** of a bond.



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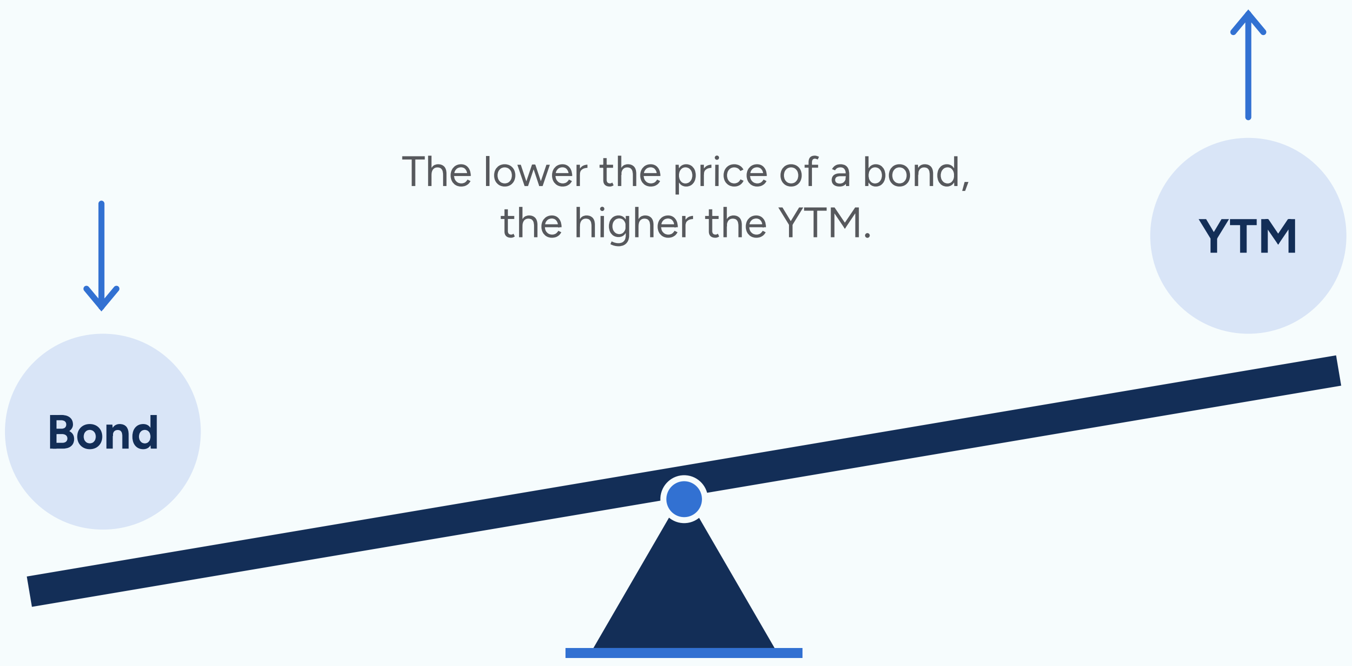


The Relationship Between Price and YTM

There is an inverse relationship between the **price** of a bond and its **yield** to maturity.

YTM	Coupon	Price	Value
6%	5%	$95.79	Discount
5%	5%	$100.00	Par
4%	5%	104.45	Premium

Using Excel to Price a Bond

Let's explore calculating **present value (PV)** using five inputs.

PV Input	Coupon	Bond Parameter
Rate	6%	YTM Per Period
NPER	5	Number of Periods
PMT	5	Coupon ($)
FV	100	Face Value ($)
Type	0	End of Period

Using Excel to Find the Yield to Maturity

Now, let's look at finding the **YTM** of an annual bond using the **RATE function** and its five inputs.

Rate Input	Value	Bond Parameter
NPER	5	Number of Periods
PMT	5	Coupon ($)
PV	-95.79	Current Price ($)
FV	100	Face Value ($)
Type	0	End of Period

Statistics for Financial Markets

S&P 500

Statistics describe the **performance** of a security. The **greater** the positive gradient of the line, the **higher** the mean return.

Exxon Mobil

Statistics help investors describe the **trend** of a security. While the **price fluctuated**, the trend of the stock was **positive**.

S&P 500

Volatility occurs when the price of the index **fluctuates a lot**. Statistics help describe how volatile an index has been.

Exxon Mobil and the S&P 500

Statistics help us describe how **two securities** move in **relation** to each other. This is known as their **correlation**.

Apple and the S&P 500

Securities can have a high degree of **correlation**. Statistics help investors describe the **relationship** between securities.

Measuring Returns

The **arithmetic return** assumes the price change of a security happens in one step at the end of the period.

Arithmetic returns work well when the start and end price are **not related** to each other.

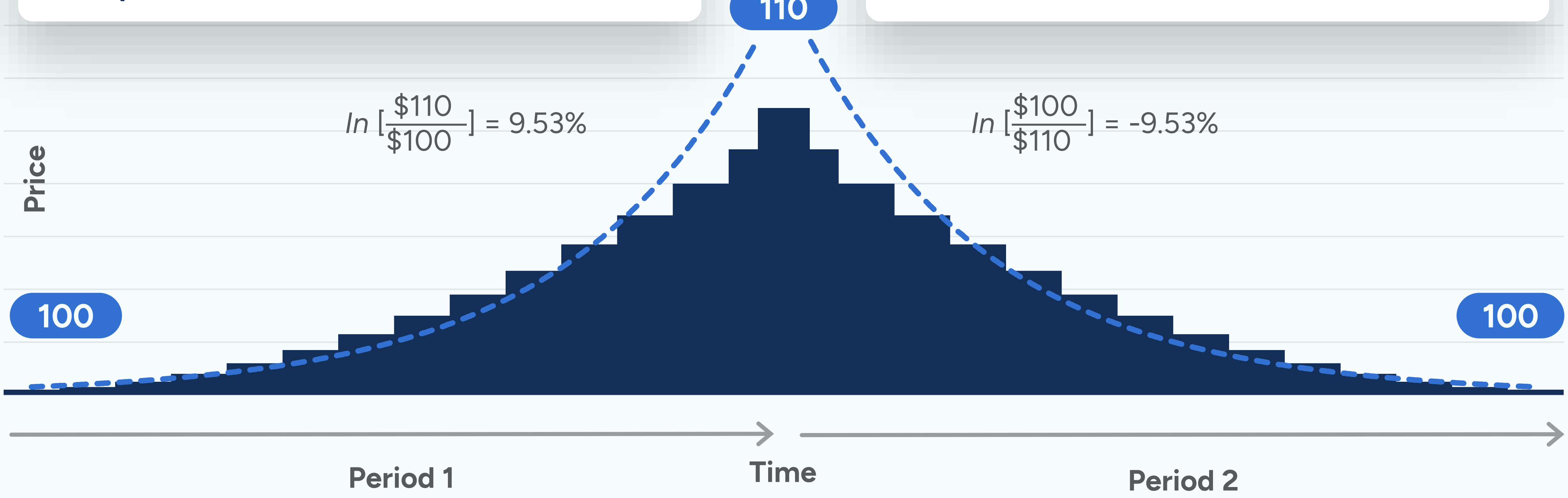
$$\frac{\$110 - \$100}{\$100} = 10.0\%$$

$$\frac{\$100 - \$110}{\$110} = -9.1\%$$

Measuring Returns

Security prices move in **lots of small steps**. So, the correct return is the **logarithmic return,** which is also called the **continuously compounded return.**

Logarithmic (log) returns work well when the start and end prices **are related** to each other.



Measuring Volatility

Volatility is a statistical measure of **spread**, or **dispersion**, of the returns of a security.

Security 1

Daily Returns →	-1%	3%	2%	-2%	3%	= 5%
Average Return →	1%	1%	1%	1%	1%	
Difference →	-2%	+2%	+1%	-3%	+2%	=0%

Difference = Daily Returns – Average Returns

Security 2

Daily Returns →	5%	-3%	7%	0%	-4%	= 5%
Average Return →	1%	1%	1%	1%	1%	
Difference →	+4%	-4%	+6%	-1%	-5%	=0%

Quoting Volatility

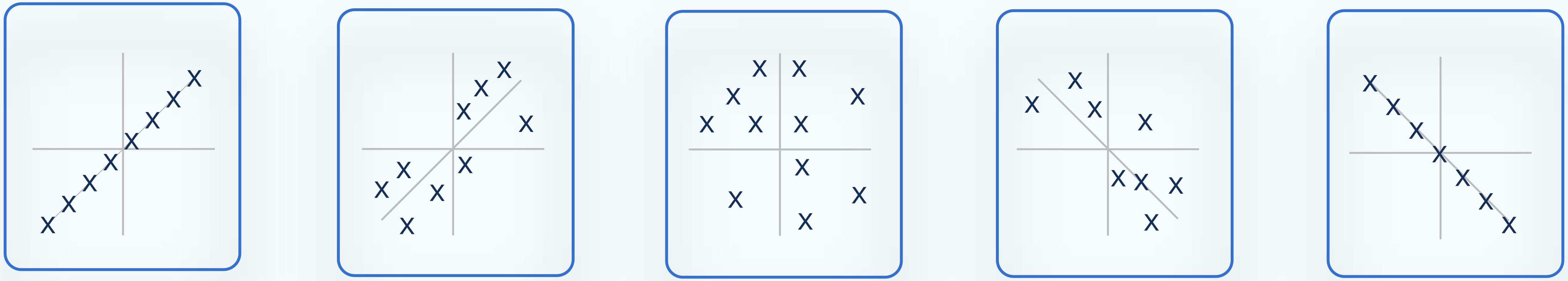
Volatility may be calculated using **daily**, **weekly,** or **monthly** data.

Volatility is always quoted on an **annual** basis in financial markets.

Frequency	Scaling Factor
Daily	$\sqrt{260}$ = 16.1
Weekly	$\sqrt{52}$ = 7.2
Monthly	$\sqrt{12}$ = 3.5

Comparing Securities

Correlation is a statistical measure of the strength of the relationship between two variables.



Perfect Positive
r = 1

Positive
r = 0.7

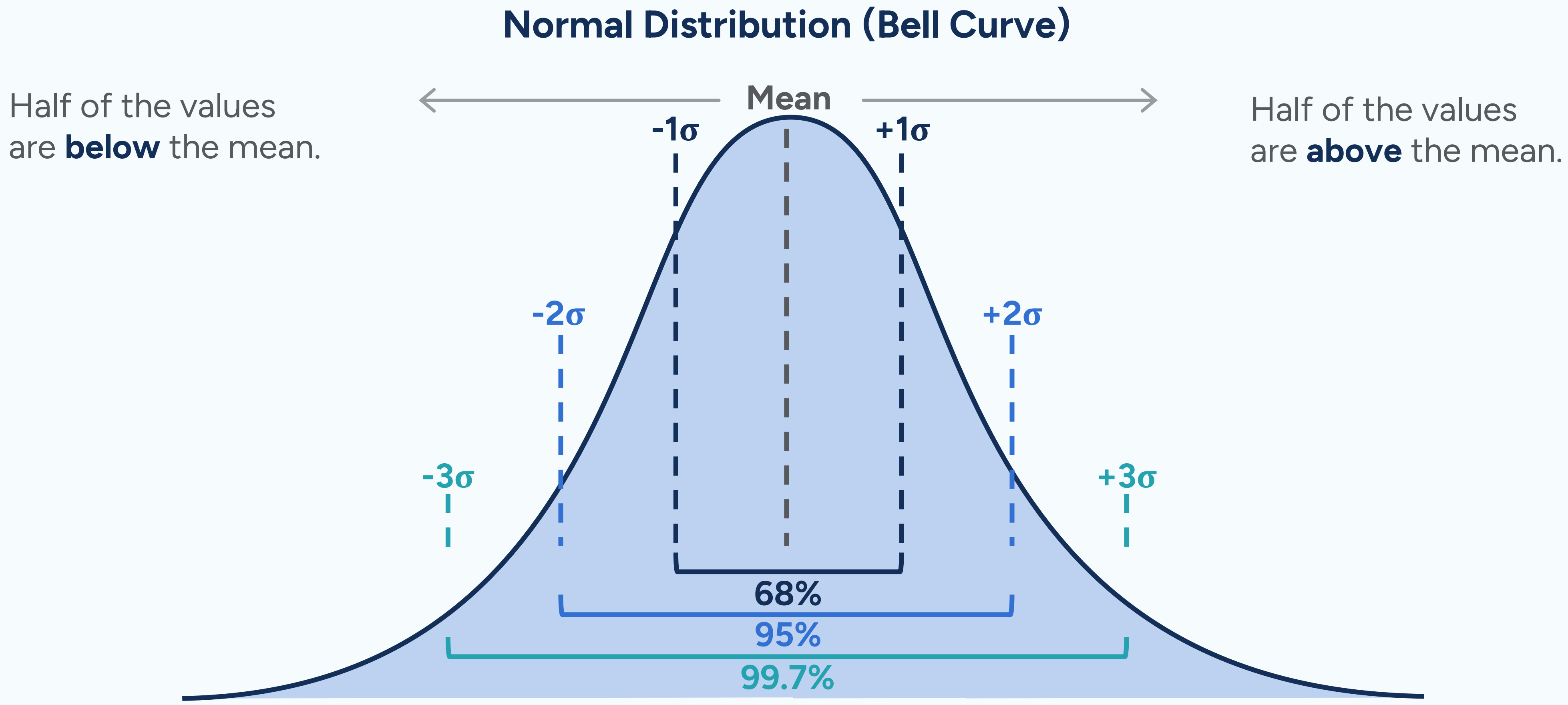
None
r = 0

Negative
r = -0.7

Perfect Negative
r = -1

The Normal Distribution

Normally distribution occurs when more values are found closer to the mean and fewer are found further from the mean.



Calculating Probabilities

Now let's explore calculating **probabilities** using **standard deviation**.

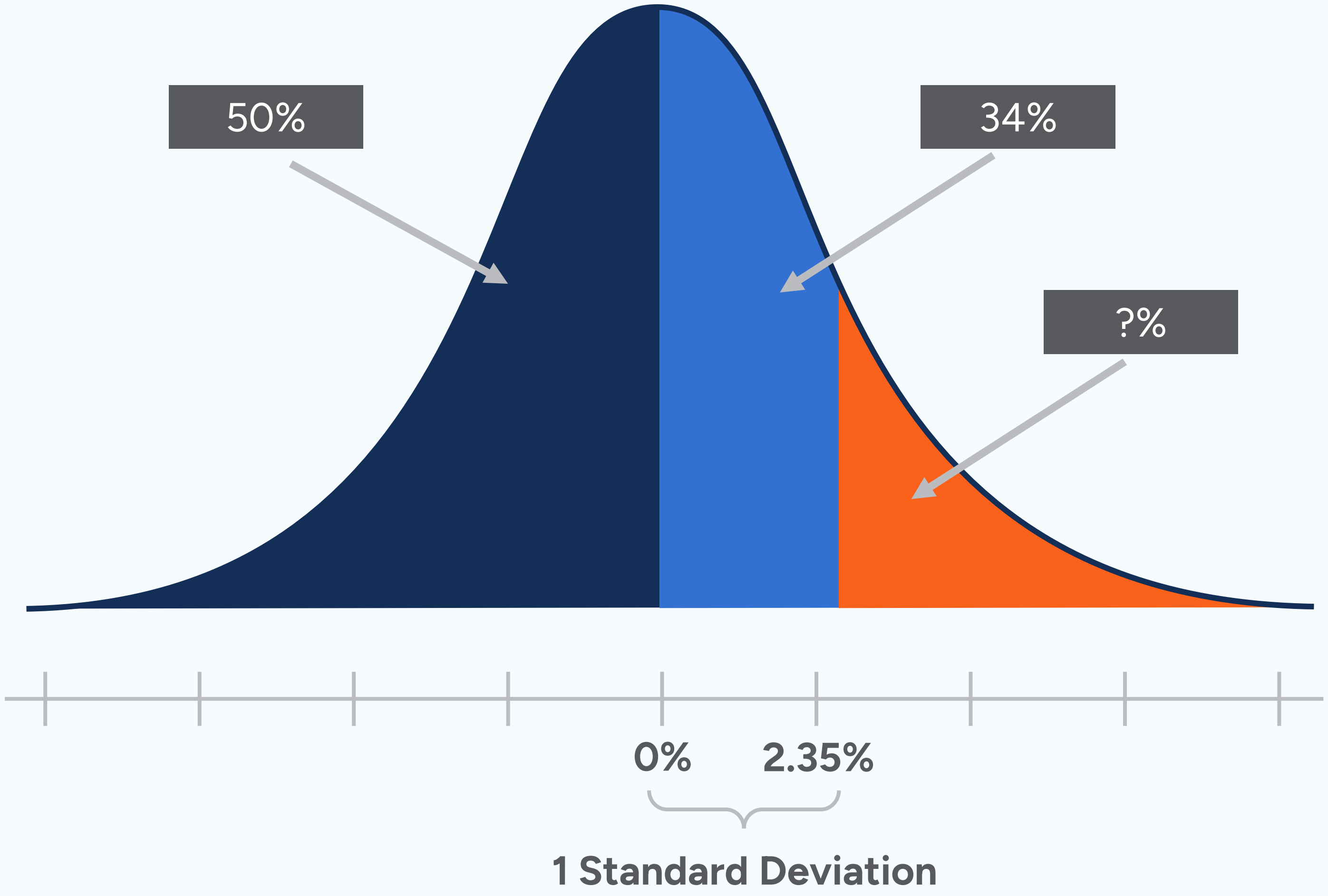
Example

Standard Deviation: 2.35%

Mean Return: 0%

Question:

What is the probability that on a given trading day, the return of the security will be higher than 2.35%?



Finding the Distance From the Mean for a Given Probability

Once we know data is **normally distributed**, we can approach the scenario from a different perspective.

Example

Daily Standard Deviation: 2.35%

Question:

How much could you lose on a bad trading day?